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Introducing a Supportive Framework to Address Students' Misconceptions and Difficulties in the Learning Mathematical Proof Techniques: A Case of Debark University in Ethiopia

Aschale Moges Belay*[a], France Machaba[b], & Tšhegofatšo Phuti Makgakga[b]

* Corresponding author
Email: 17217059@mylife.unisa.ac.za

a. Department of Mathematics, Debark University, Ethiopia

b. Department of Mathematics Education, UNISA, South Africa

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ABSTRACT

This research article is about "Introducing a Supportive Framework to Address Students' Misconceptions and Difficulties in Learning Mathematical proof techniques (MPT): A Case of Debark University". This study aims to develop, introduce, and implement a supportive framework to overcome students' misconceptions and difficulties in MPT. The framework, named IR^2CP^2CE, was developed, introduced, and implemented at Debark University in Ethiopia using various data-gathering instruments such as questionnaires, interviews, classroom observations, and document analysis from students and instructors. The study collected data over four months, including the implementation of a supportive framework using mixed, quasi-experimental, and pragmatism research approaches, designs, and paradigms respectively. The internal reliability of the data-gathering instruments was interpreted using Cronbach's coefficient, Spearman-Brown, Spearman correlations, Kuder-Richardson 20 and 21, and difficulty and discrimination indices. The results showed that the implementation of the supportive framework led to significant improvements in students' academic performance in MPT, regardless of factors such as gender, academic year category, and preliminary knowledge and proving skills. This study recommends additional imperatives for practice and future research.

KEYWORDS

Learning difficulties; learning misconceptions; mathematical proof; mathematical proof techniques; supportive framework.

INTRODUCTION

Overview of the Study

Mathematics is an abstract science that deals with quantities, numbers, and spaces, either as pure mathematics (an abstract concept) or applied mathematics (applied to other disciplines) (Wittmann, 2020). It is crucial for students to develop the cognitive, affective, and psychomotor domains of learning. Mathematics helps students represent abstract concepts in pictorial and visible ways, enabling them to correlate abstract concepts in clear and observable ways. It models phenomena mathematically and codes abstract concepts in memorable ways, providing logical reasons for the truth and proving the truthfulness of facts (Daras, 2016).

Scholars such as Piaget, Tall, and Sharp and Cole have conducted research on students' cognitive development in mathematics. They categorized cognitive development into three interrelated worlds: conceptual-embodied, perceptual-symbolic, and formal-axiomatic (Chlebowski, 2021). These three worlds are directly correlated with the mathematical proof of statements, which requires critical thinking, symbolic representation, and the formulation of formal axioms and properties (Wallace, 2021). The development of students' understanding is one part of their cognitive development, and mathematics has applications in developing students' levels of mathematical understanding. Richard Skemp (1979) categorized mathematical understanding into instrumental and relational understandings. Instrumental understanding involves applying formulae and rules without any rationale behind them, whereas relational understanding involves connecting formulae and definitions to build general mathematical understanding (Agustina et al., 2021).

Knowing and describing the teaching–learning styles of mathematics in Ethiopia is vital for understanding the background of existing problems in the area of mathematical proofs. The Ethiopian Ministry of Education divides the history of Ethiopian education into two parts: modern and traditional education (Tamrat, 2022). The Orthodox Church dominated traditional education, which taught subjects such as medicine, ethics, vocation, language, religion, astronomy, and mathematics. The church has made contributions to the development of mathematics in Ethiopia, such as symbolic representation for numbers of Geez, methods for arithmetic operations, mathematical algorithms for the Ethiopian calendar, and methods for constructing mathematical shapes such as Aksum monuments and Lalibela rock-hewn churches. However, traditional education in Ethiopia has limitations such as not being secular, using traditional teaching methodologies, and not representing numbers beyond natural numbers (Shume, 2022).

Modern education in Ethiopia began during the regime of Emperor Menilek II in the early 20th century, with teachers unfamiliar with Ethiopian society and curricula design based on foreign countries. The curriculum was mainly teacher-centered, limiting students' opportunities to actively participate in the teaching-learning processes. After Emperor Haile Selassie I, the curriculum was revised to include Ethiopian contexts and allow active student participation (Argaw, 2015).

The current Ethiopian educational system is divided into four categories: preschool education (KG 1 to 3), primary education (grades 1 to 8), secondary education (grades 9 to 12), and tertiary education (college and university levels). Preschool education is limited to urban Ethiopia. Primary education consists of the lower primary level (grades 1–4) and the upper primary level (grades 5–8), and secondary education consists of the lower secondary level (grades 9–10) and the upper secondary level (grades 11–12) (Kelkay, 2023).

Ethiopian students learn mathematics from preschool to university level to achieve different goals within different credit hours. Preschool students can learn mathematics to count numbers, recognize symbols of numbers, and perform arithmetic operations using their fingers or objects found in their environments. Primary school students can learn mathematics for two hours per week from certificate- or diploma-qualified teachers. In grades 9 and 10, students develop solid mathematical skills, attitudes, and knowledge that contribute to the country's development and awareness of Ethiopia's cultural, political, social, and economic realities (Sileshi, 2022). In grades 11 and 12, learning mathematics should promote students' abilities in applying subject-general and specific mathematical methods of working and thinking. Upper secondary school courses differ content-wise for students in the natural science and social science streams. Secondary students can learn mathematics for 3.5 hours per week from Bachelor- or Master-qualified teachers. However, most teachers at colleges of teachers' education, primary schools, and secondary schools undergo pedagogical training before they are recruited as teachers, whereas instructors at universities and the College of Technique and Vocation Training do not (Wariyo, 2020).

The current curriculum of mathematics from preschool to university level in Ethiopia increases students' skills in mathematical proof using different schemes and levels. However, most primary and secondary teachers teach mathematical concepts without showing proof of the mathematical concepts, leading to low achievement in proof due to poor skills in proof construction. Teachers at colleges of teachers' education and universities do not use special and supportive teaching methods to teach proof (Shume, 2022).

In conclusion, understanding the teaching–learning styles of mathematics in Ethiopia is essential for addressing the challenges faced by students in learning mathematical proofs. By recognizing and addressing these factors, educators can better support students in their pursuit of knowledge and understanding in MPT (Kelkay, 2023). Researching misconceptions and difficulties in learning MPT in Ethiopia is crucial for creating awareness among concerned bodies and developing frameworks to support students with misconceptions and difficulties in learning MPT.

Statement of the Problem

The fundamental concept of algebra (FCA) is a branch of mathematics that studies representations of problems or situations in mathematical expressions. In Ethiopia, it is taught as a three-credit-hour-per-week course for second-year undergraduate mathematics department students in the second semester. The central goal of this course is to maximize students' critical thinking and reasoning skills for advanced mathematics courses. One chapter in the FCA focuses on MPT (Wesley, 2018). Students at Debark University in Ethiopia have shown a decrease in their FCA scores from 2019 to 2023. This is because of their serious misconceptions and difficulties in learning MPT. This problem leads to students' inability to effectively and efficiently learn other higher mathematics courses and related disciplines.

No research has been conducted on developing a supportive framework for students with misconceptions and difficulties in learning MPT in the context of Ethiopia (Argaw, 2015; Minister, 2020; Molla, 2018; Sileshi, 2022; Wagaw, 2018). Stefanowicz (2021) states that the reviewed literature focuses more on mathematical content, students' academic level, and only a single technique of mathematical proof. However, there is a major gap in the literature on developing a supportive framework for students with misconceptions and difficulties in learning MPT (Abraham et al, 2017; Shume, 2022).

LITERATURE REVIEW

This study's literature review explored various topics related to mathematical proof, learning difficulties, misconceptions, factors influencing students' understanding, theoretical framework, and mechanisms to support students with misconceptions and difficulties in learning MPT.

Mathematical Proof

Mathematical proofing is the process of demonstrating the truthfulness of a certain fact using various techniques and approaches in mathematics learning. This is an inferential argument that shows that the stated assumptions (premises) of a mathematical statement logically guarantee its conclusion (Loehr, 2019). The skeleton of mathematical proofing has three components: a hypothesis, a conclusion, and constructed ideas (Arbaugh et al., 2018). Pythagoras stated that mathematical proofs verify the truthfulness of a statement, explain why a statement is true, systematize findings using concepts, axioms, and theorems, discover new findings, transmit mathematical concepts, and provide intellectual challenges (Hanna et al., 2009).

Schemes of students' proof fall into three categories: external-oriented, empirical, and analytical schemes of proof (Erickson & Lockwood, 2021). External-oriented schemes accept the validity of an argument or build arguments based on an authority's word, ritual, or symbolic manipulation. Empirical schemes can be perceptual or inductive, with perceptual schemes validating conjectures through rudimentary mental images, whereas inductive schemes use quantitative evaluations. Analytical schemes validate conjectures through logical deduction reasoning and can be axiomatic or transformational (Contay & Duatepe, 2018; Guler & Sen, 2015).

Herawati and Netti (2019) state that a mathematical proof has three levels: pragmatic, intellectual, and demonstrative. The lowest level is the pragmatic level of proof, which is representation with examples; the medium level is the intellectual level of proof, constructed based on formulation; and the highest level is the demonstrative level of proof, organized by a theory or community-accepted knowledge. Loehr (2019) defines techniques for mathematical proof as those that can be used to prove the truthfulness of given mathematical statements. There are many techniques for mathematical proof or disproof, including combinatorial proof (CP), direct proof (DP), disproof by counter-examples (DCE), probabilistic proof (PP), proof by construction (PCS), proof by contradiction (PCD), proof by contrapositive (PCP), proof by exhaustion (PE), proof by mathematical induction (PMI), and proof by using rules of inference (PRI).

Proof by contrapositive proves a mathematical statement by starting with its contrapositive; proof by mathematical induction uses the induction of mathematics to prove a mathematical statement; and direct proof uses the premise of a mathematical statement as the true statement to prove its conclusion. A mathematical proving method called disproof by counterexample involves thinking up and taking counterexamples to falsify a statement. Proof by construction demonstrates a mathematical statement by creating counterexamples that fulfill the property. Proof by exhaustion uses infinite cases to prove mathematical statements; probabilistic proofs use probability theory to prove mathematical statements, while proof by contradiction starts with the negation of a statement. Combinatorial proof uses mathematical counting and combination to prove a mathematical statement, and proof using the rules of inferences uses references to prove mathematical statements. (Garnier & Taylor, 2016; Hamkins, 2021; Reiser, 2020).

Learning Difficulties

Dyscalculia is a learning difficulty in mathematics that involves problems in applying mathematical principles and understanding the meaning of numbers. Students with dyscalculia struggle with memory retention, demonstrate impulsive problem-solving, inaccurate recall of basic arithmetic facts, poorly developed number senses, and mental representation of mathematical concepts. They face difficulties in learning calculations, engaging in daily activities, and providing logical reasons for facts (Cusack, 2021). Students with dyscalculia also face difficulties in learning MPT, which may hinder their ability to understand mathematical statements and construct proofs. According to Arbaugh et al. (2018), students' difficulties in learning the techniques of mathematical proof mean that they face obstacles in learning, understanding, and constructing proofs that show the truthfulness of mathematical statements using different techniques. Students' proof difficulties are categorized into two categories: lack of conceptual knowledge required to complete a proof and lack of knowledge about systematic methods used in proofs (Arana & Stafford, 2023).

There are six indicators to determine students' difficulties in mathematical proof: constructing the proof skeleton, making the skeleton of the proof, understanding and reading the proof, knowing the goals of the proof, providing illustrative examples, and coordinating mathematical statements with preliminary concepts (Abadi et al, 2019; Mtsem & Opadeyi, 2020). According to Mononen and Rong (2022), collaboration among students, teachers, parents, and educational experts can help minimize learning difficulties in mathematics

Learning Misconceptions

Mathematical knowledge is summative and requires a direct correlation between newly developed and old knowledge (Mononen & Rong, 2022). Misconceptions can occur when the correlation is weak. Scientific misconceptions are mainly focused on beliefs about science that are not supported by scientific evidence. These misconceptions can be categorized into five main categories: factual, conceptual, non-scientific, vernacular, and preconceived notions (Kinchin, 2019). According to Becker (2019), students' misconceptions about learning certain mathematical content vary on the basis of the content's solidity, with serious misconceptions about mathematical proof being more common among students at different educational levels (Mononen & Rong, 2022). Misconceptions can lead to errors, which can take two forms: execution and conceptual errors. Misconceptions are deeply rooted in cognitive structures and can hinder new knowledge formation.

To differentiate misconceptions from lack of knowledge, it is essential to differentiate teaching methods that correct students' misconceptions and fix a lack of knowledge. The certainty response index (CRI) measures students' certainty responses to a specific concept. A score of 5 indicates high confidence in answering questions, whereas a score of 0 indicates poor understanding or lack of knowledge (Hayati & Setyaningrum, 2019). A high level of CRI indicates that students have good knowledge of the course, whereas a low level indicates that they depend on guessing or lack proper procedures. Differentiating misconceptions from lack of knowledge involves identifying students' characteristics, such as wrong perception, lack of knowledge, lucky guesses, and knowledge of a correct concept (Hayati & Setyaningrum, 2019). Students' misconceptions about techniques of mathematical proof mean that they can have incorrect ideas and understanding of proofs that are important to show the truthfulness of mathematical statements using different techniques in the learning of mathematics (Arbaugh et al., 2018).

Factors Influencing Students' Understanding of Proof

A knowledge of factors influencing students' understanding of proof is crucial for developing supportive frameworks for those with misconceptions and difficulties in MPT. Students dislike changeable patterns in mathematical proof procedures because they prefer uniform patterns. However, developing skills to construct proofs with changeable patterns can increase abstract thinking and reasoning power (Ahmadpour et al., 2019). Students can prove mathematical statements efficiently and effectively if they have the skills to identify the hypothesis, understand the preliminary definitions, develop the structure of the proof, correlate the statement with previously learned concepts, change the statement into a formula and symbolic representation, and develop confidence in constructing the mathematical proof. Teachers' pedagogical knowledge and curriculum design also influence students' skills in constructing mathematical proof (Arbaugh et al., 2018).

The complexity or simplicity of mathematical proofs varies from technique to technique, and factors such as students' age, teachers' teaching style, beliefs, course content flow, and experience in proving can affect it. Teachers can minimize students' misconceptions and difficulties in mathematical proof by providing continuous feedback on their skills (Erickson & Lockwood, 2021).

In conclusion, the dependent variable in proof learning is influenced by three independent variables: teachers' factors, students' factors, and other factors. Teachers' factors include knowledge, teaching methods, assessment styles, and feedback. Students' factors include their attitudes, skills, experiences, and relationships with teachers. Other factors include the length of proof steps, content flow, and uniformity of patterns (Adhikari, 2021).

THEORETICAL FRAMEWORK OF THE STUDY

This section presents theories supporting research activities, primarily learning theory and the philosophy of mathematics, as theoretical frameworks for research tasks and activities.

Learning Theory

Learning is a permanent modification of behavior resulting from practices or experiences. There are three main learning theories: behaviorism, cognitivism, and constructivism. Behaviorism believes that learning occurs through teachers' punishments and rewards, whereas cognitivism focuses on how information is received, organized, stored, and retrieved by the mind. Constructivism, on the other hand, believes that learners can construct new learning ideas based on their prior knowledge and experiences (Illeris, 2018; Johnson, 2019).

Mononen and Rong (2022) emphasize that misconceptions and difficulties in learning mathematics concepts occur when learners do not construct new knowledge based on well-constructed previous knowledge. This research focused on and used constructivism learning theory, which has three parts, including social constructivism. Social constructivism, developed by Lev Vygotsky in 1968, views learning as occurring through social interaction and the help of others, often in a group. Vygotsky emphasizes the importance of language and culture in human intellectual development and perception of the world.

This study applied social constructivism and scaffolding in the implementation of supportive frameworks. The teaching methodology in social constructivism classes is student-centered and involves active student participation in teaching–learning activities. This student-centered teaching methodology is crucial for solving students' misconceptions and difficulties in learning techniques for mathematical proof.

In conclusion, social constructivism, as described by Vygotsky, was the theoretical framework for this research. The principles of the more knowledgeable other (MKO), scaffolding, and the zone of proximal development (ZPD) guided the research to effectively and efficiently accomplish its activities.

Philosophy of Mathematics

The philosophy of mathematics is a branch of philosophy that studies the philosophical assumptions, foundations, and implications of mathematics. It focuses on understanding mathematical truth, proof, evidence, practice, and explanation. There are five branches of the philosophy of mathematics: logicism, Platonism, formalism, fallibilism, and intuitionism (Linnebo, 2020). Logicism posits that mathematics is an extension of logic, whereas Platonism posits that abstract mathematical objects exist independently of human language, thought, and practices. Formalism argues that all mathematics can be reduced to formula manipulation rules without referencing their meanings. Fallibilism asserts that empirical knowledge cannot be achieved completely, whereas intuitionism posits that self-evident laws govern mathematical discourse (Cevik, 2021). This research used the philosophy of mathematics as a theoretical framework, considering all five branches to address misconceptions and difficulties in learning MPT.

Mechanisms to Support Students with Misconceptions and Difficulties in Learning MPT

The supportive framework is a structured approach designed to address a student's existing problem by providing strategies to minimize it (Loehr, 2019). It follows a problem-solving method that involves identifying the problem, identifying its causes, designing a solution, implementing the solution, and evaluating its effectiveness. This approach provides a more effective and efficient solution for students. The framework, named IR^2CP^2CE (Identify, Record, Report, Create, Prepare, Create, and Examine), is crucial for students struggling with misconceptions and difficulties in learning MPT. IR^2CP^2CE was developed using the stated literature, theoretical frameworks, and data gathered from students and instructors using the data collection instruments of this study (that means RQ 1 has been answered) and it has the following procedures.

Stage 1: Identifying misconceptions and difficulties of each student in learning MPT.

Stage 2: **Recording** the identified misconceptions and difficulties of each student in learning MPT.

Stage 3: Reporting the recorded misconceptions and difficulties of each student in learning the MPT.

Stage 4: **Creating** discussions with students, staff members of the mathematics department, the mathematics department head, and the College of Natural and Computational Science dean at Debark University.

Stage 5: **Preparing** a plan to accomplish the teaching–learning processes for the students.

Stage 6: **Preparing** a handout with the title "MPT".

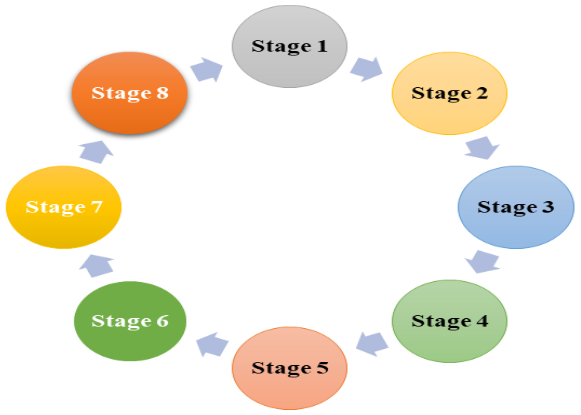
Stage 7: **Creating** a special class for the students. Theories such as Bloom's taxonomy, social constructivism, ZPD, MKO, scaffolding, and the philosophy of mathematics guided the special class. It involved nine proving steps: reading, identifying, symbolizing, memorizing, thinking deeply, selecting the appropriate MPT, starting the proof process, stating supportive reasoning ideas, and checking the proof for mistakes (Anerk et al, 2020; Fishirty et al, 2019; Jones, 2020). These steps are crucial for proving mathematical statements accurately and effectively. The class approach ensured a comprehensive understanding of the mathematical concepts.

Stage 8: Examining whether or not the identified misconceptions and difficulties have been avoided.

The supportive framework is cyclic, repeating stages 1-8 until the problem is resolved, as illustrated in the following diagram.

Figure 1

The Developed Supportive Framework



Research Questions

This study addressed four key questions based on the concepts discussed in the Introduction and Literature Review sections.

1. What are the mechanisms that support students with misconceptions and difficulties in learning MPT?
2. What are the effects of IR^2CP^2CE on overcoming students' difficulties and misconceptions in learning MPT?

Objectives of the Study

This study mainly aimed to introduce, develop, and implement a supportive framework to overcome students' misconceptions and difficulties in learning MPT. It is named IR^2CP^2CE (Identifying, Recording, Reporting, Creating, Preparing, Creating, and Examining).

RESEARCH METHODOLOGY AND DESIGN

Research approach, design, and paradigm

This study used a mixed research approach to collect and analyze both qualitative and quantitative data on the supportive framework for students' misconceptions and difficulties in learning MPT. It employed a quasi-experimental design, dividing the study into control and experimental groups. This study aimed to determine whether developed supportive frameworks can improve existing problems, with a pragmatism paradigm being used because

Table 1

The Demographic Information of Control and Experimental Groups

Students' category	CG			EG		
	Male	Female	Total	Male	Female	Total
Third year	7	2	9	8	1	9
Fourth year	5	1	6	4	2	6
Total	12	3	15	12	3	15

of the mixed approach. Table 1 shows the demographic information of the control group (CG) and experimental group (EG).

Sources of Data

This research used data from third- and fourth-year mathematics department students, instructors, and student documents at Debark University, Ethiopia, to understand misconceptions and difficulties in learning MPT and to provide insights for overcoming these issues.

Data Gathering Instruments

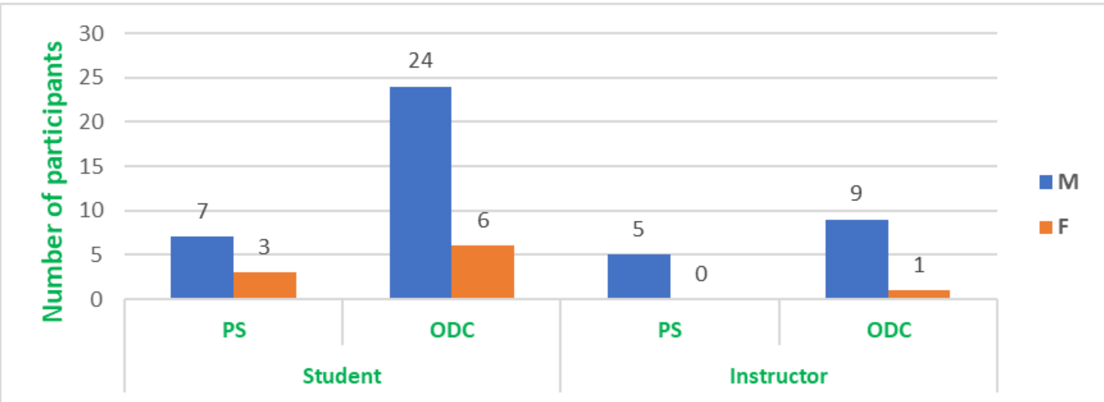
This study used several data-gathering instruments, such as questionnaires for students and instructors, interviews with students, classroom observations, a certainty response index of students' pretests, and document analysis of students' tests and assignments, to collect data from the research subjects.

Sample Sizes and Sampling Technique

The Mathematics Department is a department at the College of Natural and Computational Science at Debark University in Ethiopia. At the time of the study, this department was providing teaching courses in undergraduate programs. The following figure shows the cumulative description of the participants' demographic information in the pilot study (PS) and the original data collection (ODC).

Figure 2

Cumulative Description of Participants' Demographic Information



The research used a simple random sampling technique to select subjects from the instructors' and students' groups at the department and a purposive sampling technique to select members of the control and experimental groups because the researchers needed their judgments to incorporate low, medium, and high academic achievement students in both the control and experimental groups.

Data Analysis and Interpretation Methods

This study used content analysis, thematic analysis, narrative analysis, grounded theory analysis, and discourse analysis to analyze qualitative data collected from the research subjects.

Descriptive and inferential statistics were used to analyze the quantitative data collected from the participants. The data were analyzed and interpreted using various statistical methods, including frequency distribution tables, figures, and t-tests, using the Statistical Package for the Social Sciences (SPSS). Therefore, the collected data were analyzed and interpreted both qualitatively and quantitatively because a mixed research approach was used in this research.

Strategies for Ensuring the Validity and Reliability of the Research

Pilot studies, experts' criticism, constructing tests using test specifications, including external colleagues in the classroom observation, constructing questionnaires using their preparation guidelines, selecting relevant and appropriate data collection instruments, sample sizes, sampling techniques, research design and research approach, applying relevant statistical techniques for data analysis, collecting and analyzing the data thoroughly, and using data triangulation (the use of two or more data collection instruments) were used to ensure the accuracy and consistency of the research data gathering instruments (Shimizu, 2022).

In addition, validity and reliability were ensured by selecting the data gathering instruments, sample sizes, and sampling techniques based on the students' backgrounds, the constructed research questions, the intended research objectives and aims, and all concepts of validity and reliability stated earlier in this section; using appropriate statistical techniques to analyze the collected data such as t-test, etc., and to check the internal consistency of data gathering instruments such as Cronbach's coefficient, Spearman-Brown, Spearman correlations, Kuder-Richardson 20 and 21; applying data triangulation; and methodologically collecting, analyzing, and interpreting the data.

RESULTS AND DISCUSSION

The research design is quasi-experimental, with control and experimental groups selected non-randomly based on pre-test scores. The experimental group was taught MPT using a supportive framework, whereas the control group learned without treatment for ten weeks to answer RQ 2. The overall implementation of the supportive framework is discussed as follows.

The *initial stage* involved identifying students' misconceptions and difficulties in learning MPT through various methods, including questionnaires, pre-tests, interviews, assignments, and classroom observations.

Misconceptions of students in learning MPT include starting with an inappropriate statement, using ineffective MPT, providing incorrect symbolic representation, providing unacceptable reasons for each step, reaching the conclusion without showing necessary steps, using non-sequential steps, incorrectly using technical aspects of mathematics, using the premise and conclusion parts interchangeably, and misusing the pattern in the proof of a certain statement for the proof of another statement. These are identified using data gathering instruments of this study.

Students face difficulties in learning MPT, including challenges in understanding the given statement, determining the MPT, developing strategies, separating the premise and conclusion parts of a mathematical statement, knowing the purpose of the proof, completing all proof steps, symbolizing statements, providing acceptable reasons, reading and understanding the proof, and lacking preliminary concepts such as definitions, properties, examples, and axioms. These are identified using data gathering instruments of this study.

In the *second stage*, students' misconceptions and difficulties in learning MPT were documented in a portfolio. Each student had a file containing their results before, during, and after the implementation of a supportive framework, feedback on teaching-learning processes, and home and class work.

In the *third stage*, students were informed about their misconceptions and difficulties in learning MPT in a written form. The goal was to help them understand the severity of these issues and encourage them to actively participate in overcoming the problem.

The *fourth stage* involves conducting discussions with students, staff, and the head of the mathematics department at Debark University. The aim of this stage is to raise awareness of the severity of the problem and its solutions and to encourage participation in action. Educational training workshops were held to further educate and empower the affected bodies to act.

Argaw (2015) emphasizes the importance of educational training and workshops in overcoming students' misconceptions and difficulties in learning mathematical concepts, including proof techniques. These strategies help in forming consensus, providing insights into problem severity, building psychological readiness, and developing strong relationships among concerned parties. As a strategy in the fourth stage of supportive frameworks, educational training and workshops can help overcome students' misconceptions and difficulties in learning MPT.

A psychology instructor and Aschale conducted educational training at Debark University in Ethiopia on June 07, 2023, for third- and fourth-year mathematics department students. The training focused on understanding the causes and mechanisms of students' misconceptions and difficulties in learning MPT, their impact, psychological readiness for overcoming these misconceptions and difficulties, and the proposed supportive framework for the problem. Students appreciated the training, as it helped them understand misconceptions and difficulties in learning MPT. They were interested in participating in problem-solving activities and showed psychological readiness. This supportive framework can be continued to tackle other mathematical concepts.

A workshop was held at Debark University in Ethiopia on June 09/2023, involving third- and fourth-year mathematics students (only in the experimental group), instructors, department heads, and academic affairs officers. The workshop focused on the impact of students' misconceptions and difficulties in learning MPT on their educational qualities. It discusses educational qualities in MPT, mechanisms for ensuring their educational value, students' misconceptions and difficulties in learning these techniques at Debark University, the role of concerned bodies in overcoming these issues, and a proposed supportive framework for addressing these issues. The workshop members were committed to actively participating in the implementation of a supportive framework to achieve the targeted aim of the proposed framework.

In the *fifth stage*, a unit plan for teaching MPT was developed on the basis of student situations and ideas from training and workshops. The plan provides guidelines for teachers to effectively and efficiently perform the teaching-learning process of a course unit, incorporating domains such as Bloom's Taxonomy of Learning, philosophy of mathematics, and social constructivism.

In the *sixth stage*, the instructor prepared teaching materials such as handouts, worksheets, and teaching aids based on the unit plan and guidelines. The preparation began in January 01/2023 to facilitate the implementation of the supportive framework. The final task in this stage was to finalize the preparation of the materials. The handout, titled "MPT," was distributed to third- and fourth-year students at Debark University in Ethiopia. The handout includes three sections: preliminaries for mathematical proof, mathematical proof, and MPT.
The worksheet contains 66 questions, including twenty questions from Section 1, sixteen questions from Section 2, and thirty questions from Section 3. The teaching aids were prepared in various forms, such as charts and videos.

The goal of this stage was to provide students with a comprehensive understanding of MPT. This stage ultimately aimed to provide a comprehensive and engaging learning experience for students.

In the *seventh stage*, the teaching-learning activities of MPT were conducted by dividing the subject of the study into two groups, namely, control and experimental groups. Each group has 15 students (3 females and 12 males).

The experimental group has more classes per week than the control group, aiming to address students' misconceptions and difficulties in learning MPT by exceeding the credit hours for teaching-learning activities. This variation in class sizes was a significant factor in this study. Debark University's mathematics instructors neglect to prepare lesson plans for teaching mathematical concepts, leading to misconceptions and difficulties in learning MPT. This study aimed to examine the impact of lesson plans on teaching-learning processes, focusing on the experimental group, as the control group's teaching– learning processes were conducted in the usual manner.

The experimental group had higher credit hours for teaching-learning activities, used multiple methods, assessment techniques, and teaching aids, and was directed by a constructed lesson plan and schedule. They provided strategies for proving mathematical statements using MPT and allowed students to prepare themselves before learning by reading handouts and doing worksheets at home.

In the *eighth stage*, the implementation of the supportive framework was evaluated using different mechanisms. One method to determine whether the implementation of the supportive framework was effective or not was through formative assessments such as classwork, homework, quizzes, oral questions, and presentations. The same types of formative assessments were administered to the control and experimental groups. The performance of formative assessments for the experimental group was recorded as a better achievement than the performance of formative assessments for the control group. This indicates that students' misconceptions and difficulties in learning can be minimized through the effectiveness of the supportive framework. Therefore, the implementation of the supportive framework was evaluated as good by considering the performance of the formative assessments for the control and experimental groups. To determine whether the implementation of the supportive framework was effective or not, the results of the control and experimental groups in summative assessments, such as the pretest and posttest, were analyzed and interpreted using a paired t-test, which was performed using Microsoft Excel and SPSS.

Evaluation of a Supportive Framework Without Considering Cases

Table 2

Results of the Control and Experimental Groups in the Pretest and Posttest

Control group					Experimental group				
SN	Sex	Age	Results in pretest	Results in posttest	SN	Sex	Age	Results in pretest	Results in posttest
2	F	1	4.7	5.75	1	F	1	4.45	10.75
4	M	1	4.75	5.25	3	F	2	6.9	11.5
5	M	2	5.3	6.5	6	M	1	11.15	14.25
9	M	2	4.55	7.75	7	M	2	11.3	15
10	M	1	6.25	8.75	8	M	1	12.3	14
12	M	2	10.35	9	11	M	1	5.85	10.25
14	F	1	5.7	6.5	13	F	1	5.7	11
15	F	1	6.1	5.75	17	M	1	7.1	14
16	M	2	5.8	6.25	19	M	2	4.8	10.75
18	M	1	11.05	9	21	M	2	4.75	11.5
20	M	1	12.75	9	22	M	1	11.7	14
23	M	1	8.3	8.5	25	M	1	5.35	10.5
24	M	1	5.65	5	27	M	1	5.95	12.25
26	M	1	7.8	7	28	M	2	5.75	10
30	M	2	14.05	12	29	M	1	7	13

Table 2 displays the results of the pretest and posttest conducted on the control and experimental groups before and after the implementation of the supportive framework. SN in the tables after this page denotes the student number. Under the age column of all tables after this page, 1, 2, 3, and 4 denote ages $18-23, 24-29, 30-35$, and 36 or above years.

The paired t-test was used to analyze the pretest and posttest results of the control and experimental groups in Table 2, and the recorded data are presented as follows.

The pretest showed a mean difference of 0.203 between the control group (M=7.54) and the experimental group (M=7.34), indicating a very similar academic background between the two groups, with no significant difference at 0.05 (P=0.854>0.05) and effect size $(\mathrm{d}) = 0.068 < 0.2$ where d is effect size. The posttest showed a mean difference of -4.717 between the control group (M=7.467) and the experimental group (M=12.183), indicating no similar academic background between the two groups, with a significant difference at 0.05 (P=2.1E-06≤0.05) and $\mathrm{d} = 2.61 > 0.2$. Students in both groups had diverse academic backgrounds in preliminary concepts and proofing mathematical statements using MPT because of the supportive framework implementation.

Hence, the implementation of the supportive framework resulted in significant improvement in the preliminary concepts and proving skills for the proof of mathematical statements using MPT.

Evaluation of the Supportive Framework by Considering Cases

Section-wise Evaluation

Table 3 displays the pretest and posttest results of the control and experimental groups in Section A, administered before and after the implementation of the supportive framework.

Table 3

Results of the Control and Experimental Groups in Section A of the Pretest and Posttest

Control group					Experimental group				
SN	Sex	Age	Results in pretest	Results in posttest	SN	Sex	Age	Results in pretest	Results in posttest
2	F	1	1	2	1	F	1	1	5
4	M	1	1	2	3	F	2	3	5
5	M	2	1	2	6	M	1	6	7
9	M	2	1	3	7	M	2	6	7
10	M	1	2	4	8	M	1	7	7
12	M	2	5	4	11	M	1	2	5
14	F	1	2	3	13	F	1	2	6
15	F	1	2	2	17	M	1	3	6
16	M	2	2	3	19	M	2	1	5
18	M	1	6	5	21	M	2	1	5
20	M	1	7	5	22	M	1	6	7
23	M	1	4	4	25	M	1	1	5
24	M	1	2	2	27	M	1	2	6
26	M	1	4	3	28	M	2	2	5
30	M	2	7	6	29	M	1	2	5

The paired t-test was used to analyze the pretest and posttest results of Section A in Table 3, and a comprehensive overview of the data is provided as follows:

Section A of the pretest showed a mean difference of 0.133 between the control group (M=3.133) and the experimental group (M=3), indicating a very similar academic background between the two groups, with no significant difference at 0.05 (P=0.878>0.05) and $\mathrm{d} = 0.061 < 0.2$. Section A of the posttest showed a mean difference of -2.4 between the control group (M=3.333) and the experimental group (M=5.733), indicating no similar academic background between the two groups, with a significant difference at 0.05 (P=5.8E-05≤0.05) and $\mathrm{d} = 2.17 > 0.2$. Students in the two groups had different academic backgrounds in the preliminary concepts for the proof of mathematical statements using MPT because of the implementation of the supportive framework.

Hence, the implementation of the supportive framework greatly improved the preliminary concepts for the proof of mathematical statements using MPT.

Table 4 displays the results of the control and experimental groups in Section C of the pretest and Section B of the posttest, administered before and after the supportive framework implementation, respectively.

Table 4

Results of the Control and Experimental Groups in Section C of the Pretest and Section B of the Posttest

Control group					Experimental group				
SN	Sex	Age	Results in pretest	Results in post-test	SN	Sex	Age	Results in pretest	Results in post-test
2	F	1	3.7	3.75	1	F	1	3.45	5.75
4	M	1	3.75	3.25	3	F	2	3.9	6.5
5	M	2	4.3	4.5	6	M	1	5.15	7.25
9	M	2	3.55	4.75	7	M	2	5.3	8
10	M	1	4.25	4.75	8	M	1	5.3	7
12	M	2	5.35	5	11	M	1	3.85	5.25
14	F	1	3.7	3.5	13	F	1	3.7	5
15	F	1	4.1	3.75	17	M	1	4.1	8
16	M	2	3.8	3.25	19	M	2	3.8	5.75
18	M	1	5.05	4	21	M	2	3.75	6.5
20	M	1	5.75	4	22	M	1	5.7	7
23	M	1	4.3	4.5	25	M	1	4.35	5.5
24	M	1	3.65	3	27	M	1	3.95	6.25
26	M	1	3.8	4	28	M	2	3.75	5
30	M	2	7.05	6	29	M	1	5	8

The paired t-test was used to analyze the results of the control and experimental groups in Section C of the pretest and Section B of the posttest. The recorded data are as follows:

Section C of the pretest showed a 0.07 mean difference between the control group (M=4.407) and the experimental group (M=4.337), indicating a very similar academic background between the two groups, with no significant difference at 0.05 (P=0.784>0.05) and $\mathrm{d} = 0.08 < 0.2$. Section B of the posttest showed a mean difference of -2.317 between the control group (M=4.133) and the experimental group (M=6.45), indicating no similar academic background between the two groups, with a significant difference at 0.05 (P=6.9E-07≤0.05) and $\mathrm{d} = 2.46 > 0.2$. Students in the two groups had different academic backgrounds in their skills to prove mathematical statements using MPT because of the implementation of the supportive framework.

Hence, the implementation of the supportive framework resulted in great improvement in students' skills in proving mathematical statements using MPT.

Gender-wise Evaluation

The paired t-test analysis of the pretest and posttest results of the control and experimental groups, considering gender, was conducted using Table 2 and stated as follows.

The pretest showed a mean difference of -0.183 between females in the control group $(\mathrm{M} = 5.5)$ and females in the experimental group $(\mathrm{M} = 5.683)$, indicating a very similar academic background between the two groups, with no significant difference at 0.05 (P=0.754>0.05) and $\mathrm{d} = 0.182 < 0.2$. The pretest also showed a mean difference of 0.3 between males in the control group ($\mathrm{M} = 8.05$) and males in the experimental group ($\mathrm{M} = 7.75$), indicating a very similar academic background between the two groups, with no significant difference at 0.05 ($\mathrm{P} = 0.832 > 0.05$) and $\mathrm{d} = 0.096 < 0.2$.

The posttest showed a mean difference of -5.083 between females in the control group ($\mathrm{M} = 6$) and females in the experimental group ($\mathrm{M} = 11.083$), indicating no similar academic background between the two groups, with a significant difference at 0.05 ($\mathrm{P} = 2.7\mathrm{E} - 04 \leq 0.05$) and $\mathrm{d} = 12.449 > 0.2$. Female students in the two groups had different academic backgrounds in the preliminary concepts and proving skills for the proof of mathematical statements using MPT because of the implementation of the supportive framework. The posttest showed a mean difference of -4.625 between males in the control group ($\mathrm{M} = 7.833$) and males in the experimental group ($\mathrm{M} = 12.458$), indicating no similar academic background between the two groups, with a significant difference at 0.05 ($\mathrm{P} = 1.02\mathrm{E} - 04 \leq 0.05$) and $\mathrm{d} = 2.45 > 0.2$. Male students in the two groups had different academic backgrounds in the preliminary concepts and proving skills for the proof of mathematical statements using MPT because of the implementation of the supportive framework.

Hence, the implementation of the supportive framework resulted in significant improvement in the preliminary concepts and proving skills for the proof of mathematical statements using MPT while considering students' gender.

Academic Year Category-wise Evaluation

The paired t-test analysis of pretest and posttest results between the control and experimental groups, considering the students' academic year categories, was conducted using Table 2.

The pretest showed a mean difference of 0.122 between third-year students in the control group ($\mathrm{M} = 8.578$) and third-year students in the experimental group ($\mathrm{M} = 8.456$), indicating similar academic backgrounds between the two groups, with no significant difference at 0.05 ($\mathrm{P} = 0.76 > 0.05$) and $\mathrm{d} = 0.044 < 0.2$. The pretest showed a mean difference of -0.675 between fourth-year students in the control group ($\mathrm{M} = 7.983$) and fourth-year students in the experimental group ($\mathrm{M} = 8.658$), indicating similar academic backgrounds between the two groups, with no significant difference at 0.05 ($\mathrm{P} = 0.28 > 0.05$) and $\mathrm{d} = 0.2$.

The posttest showed a mean difference of -4.222 between third-year students in the control group (M=7.667) and third-year students in the experimental group (M=11.889), indicating no similar backgrounds between the two groups, with a significant difference at 0.05 ($\mathrm{P} = 0.001 \leq 0.05$) and $\mathrm{d} = 2.26 > 0.2$. The posttest showed a mean difference of -5.458 between fourth-year students in the control group ($\mathrm{M} = 7.167$) and fourth-year students in the experimental group ($\mathrm{M} = 12.625$), indicating no similar academic background between the two groups, with a significant difference at 0.05 ($\mathrm{P} = 0.002 \leq 0.05$) and $\mathrm{d} = 3.008 > 0.2$.

Hence, the implementation of the supportive framework resulted in significant improvement in the preliminary concepts and proving skills for the proof of mathematical statements using MPT while considering students' academic year category.

The study found that despite the control and experimental groups having similar academic levels before the treatment, the implementation of a supportive framework led to significant academic improvement in students, regardless of factors such as gender, academic year category, and preliminary knowledge and proving skills for mathematical statements using MPT.

The implementation of the IR^2CP^2CE supportive framework at Debark University significantly improved students' misconceptions and difficulties in learning MPT. This was achieved through tasks such as identifying misconceptions, recording them, reporting them, and creating discussions with students, staff, and the college dean. The framework also included the preparation of handouts, worksheets, and teaching aids to address various issues. The implementation of the supportive framework was successful; therefore, another supportive framework was not designed and implemented. The IR^2CP^2CE framework significantly improved students' understanding of MPT, demonstrating the effectiveness of the supportive framework in addressing students' misconceptions and difficulties.

CONCLUSION

Based on the information presented above, the study can draw the conclusion that even though the control and experimental groups had the same academic level before the treatment that was provided to the students, the implementation of the supportive framework brought students' academic improvement without considering any cases, and with considering cases such as their gender, academic year category, preliminary knowledge, and proving skills for the proof of mathematical statements using MPT. This conclusion was reached although the two groups had the same academic level before the treatment that was provided to the students. This improvement was made possible because of the confounding variables that were controlled, such as the tasks that were performed to identify misconceptions and difficulties of each student in learning MPT, record the identified misconceptions and difficulties of each student in learning MPT, report the recorded misconceptions and difficulties of each student in learning MPT to each student, and create discussions with students, staff members, and other participants. The study did not continue to implement another created supporting framework because the developed supportive framework's implementation was successful and the study did not want to undermine that success. The introduction of the IR^2CP^2CE supporting framework led to an improvement in both the students' misperceptions and their difficulties in acquiring MPT at Debark University, which resulted in the improvement of both areas.

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